steady loading. However, since resonance depends on the propagation of unsteady disturbances into the far field and it is not clear that a linear supersonic formulation can accurately simulate far-field behavior, the linear model may not be adequate for actual cascades or fans operating near predicted resonance conditions. The present author disagrees with the contention expressed in Ref. 2 that a linear analysis using a passage approach can be used to circumvent this problem. In a passage approach the blade-to-blade periodicity requirement must be supplemented by prescribing information on an upstream boundary of a given blade passage which must be satisfied by the velocity potential. Such information depends on the unsteady disturbances generated by the infinite array of blades below the reference passage. If the potential distribution on the prescribed upstream boundary is determined on the basis of linear equations, as in Ref. 4, and the linear formulation possesses a unique solution, then resonant points should be predicted in the final solution.

It appears that the only way to resolve the resonance problem is to introduce nonlinear effects into the governing equations in an attempt to model far-field behavior properly. This has been done for the case of steady supersonic flow past a thin, isolated airfoil. Although Ackeret's classical linearized solution is a proper first approximation in the near field, it fails at great distances from the airfoil. Ackeret's solution predicts disturbances propagating undiminished along the freestream Mach lines to infinity, whereas in reality the Mach lines are not straight and parallel. In this situation the nonlinear terms are small compared with the linear ones; however, their cumulative contribution gives rise to a first-order effect as the distance from the airfoil increases. A uniformly valid first-order approximation has been achieved by adding a nonlinear term, called the "pseudo-transonic" term, to the differential equation governing the velocity potential.[5] A solution to the resulting boundary value problem, obtained by the method of strained coordinates, gives velocity components that are the same function of distance along revised Mach lines as predicted by Ackeret's solution along the freestream Mach lines.

Since the flow adjacent to a given blade of a supersonic cascade with subsonic axial flow is influenced by the far-field disturbances from the blades below, it may prove necessary to incorporate similar considerations into an unsteady supersonic cascade analysis. However, up to the present time the linear formulation has been quite successful at predicting the measured flutter behavior of supersonic test fans.[6,7] Of course this success could be due to the inherent stability of these fans when operating near predicted resonance and further experimental work is necessary for clarification. The introduction of nonlinear terms into an unsteady analysis would lead to complications over and above those encountered in the foregoing steady example. When nonlinear terms are included, time dependence can not be eliminated from the resulting boundary value problem by simply removing the factor $e^{i\omega t}$ from each term in the governing equations.[1] In addition to the fundamental excitation at the blade motion frequency, higher harmonic excitations must also be considered. If time dependence cannot be avoided in a nonlinear formulation, one of the major features, i.e., computational efficiency, which renders a flutter analysis useful to the designer, will be lost. Obviously the resonance issue raised by Kurosaka[2] requires some further work before a satisfactory resolution will be achieved.

References

[1] Verdon, J. M. and McCune, J. E., "Unsteady Supersonic Cascade in Subsonic Axial Flow," *AIAA Journal*, Vol. 13, Feb. 1975, pp. 193-201.

[2] Kurosaka, M., "On the Issue of 'Resonance' in an Unsteady Supersonic Cascade," *AIAA Journal,* Vol. 13, Nov. 1975, pp. 1514-1516.

[3] Samoilovich, G. S., "Resonance Phenomena in Sub- and Supersonic Flow through an Aerodynamic Cascade," *Mekhanika Zhidkosti i Gaza*, Vol. 2, May-June 1967, pp. 143-144.

[4] Kurosaka, M., "On the Unsteady Supersonic Cascade with a Subsonic Leading Edge—An Exact First-Order Theory—Parts 1 and 2," *ASME:Transactions, Ser. A – Journal of Engineering for Power*, Vol. 96, Jan. 1974, pp. 13-31.

[5] Van Dyke, M. *Perturbation Methods in Fluid Mechanics*, Academic Press, New York, 1964, pp. 106-112.

[6] Snyder, L. E. and Commerford, G. L., "Supersonic Unstalled Flutter in Fan Rotors; Analytical and Experimental Results," *ASME; Transactions, Ser. A – Journal of Engineering for Power*, Vol. 96, Oct. 1974, pp. 379-386.

[7] Mikolajczak, A. A., Arnoldi, R. A., Snyder, L. E., and Stargardter, H., "Advances in Fan and Compressor Blade Flutter Analysis and predictions," *Journal of Aircraft,* Vol. 12, April 1975, pp. 325-332.

Comment on "Bolotin's Method Applied to the Buckling and Lateral Vibration of Stressed Plates"

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DICKINSON[1] has applied Bolotin's edge effect method to vibration and buckling of rectangular orthotropic plates subjected to uniform in-plane loads. The successful estimation of buckling loads was particularly interesting to the writer, since he had once attempted the same analysis of a similar problem and found the method to be inapplicable. The purpose of this Comment is to point out that the edge effect method is not universally applicable when one or both of the in-plane loads is compressive.

It is useful to reproduce, from Dickinson's paper, the frequency equation

$$\omega^2 = (1/\rho)\,[D_x (k_x/a)^4 + 2H(k_x k_y/ab)^2 + D_y (k_y/b)^4 + N_x (k_x/a)^2 + N_y (k_y/b)^2]$$

and the equations for the parameters associated with that part of the transverse displacement that decays away from the edges,

$$\gamma_x^2 = k_x^2 + 2(H/D_x)\,(k_y a/b)^2 + N_x a^2/D_x$$

$$\gamma_y^2 = k_y^2 + 2(H/D_y)\,(k_x b/a)^2 + N_y b^2/D_y$$

For Bolotin's method to work, γ_x^2 and γ_y^2 must be positive, which is clearly the case if neither N_x nor N_y is negative (in-plane compression). If either N_x or N_y is negative it is not clear, in advance of determining the wave numbers k_x and k_y, whether or not the edge effect is, in Bolotin's[2] words, "degenerate."

If we restrict our interest to an analysis of buckling ($\omega^2 = 0$), the frequency equation may be solved for the buckling loads,

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$$N_x(k_x/a)^2+N_y(k_y/b)^2=-D_x(k_x/a)^4$$
$$-2H(k_xk_y/ab)^2-D_y(k_y/b)^4$$

For the case $N_y=0$,

$$\gamma_x^2=-(D_y/k_x^2D_x)(k_ya/b)^4$$

Consequently, the edge effect associated with an edge $x=$ constant degenerates, and the method cannot be used to estimate uniaxial buckling loads.

Dickinson computed buckling loads for the case of hydrostatic in-plane loading, $N_x=N_y$. For this case

$$\gamma_x^2=[(k_xk_ya/b)^2+(H/D_x)(2-D_y/H)(k_ya/b)^4]$$
$$/[k_x^2+(k_ya/b)^2]$$

and

$$\gamma_y^2=[(k_xk_yb/a)^2+(H/D_y)(2-D_x/H)(k_xb/a)^4]$$
$$/[(k_xb/a)^2+k_y^2]$$

Therefore, the edge effect is not degenerate if neither D_x/H nor D_y/H exceeds 2, which is precisely the largest value of these parameters for which Dickinson reported numerical results.

References

[1] Dickinson, S.M., "Bolotin's Method Applied to the Buckling and Lateral Vibration of Stressed Plates," *AIAA Journal,* Vol. 13, Jan. 1975, pp. 109-110.

[2] Bolotin, V.V., "An Asymptotic Method for the Study of the Problem of Eigenvalues for Rectangular Regions," *Problems of Continuum Mechanics,* Society of Industrial and Applied Mathematics, Philadelphia, Pa., 1961, pp. 56-68.

Reply by Author to W.W. King

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THE author is in agreement with the comments of Professor King on the fact that the edge effect method, as proposed by Bolotin, is not universally applicable to the vibration and buckling of plates under uniaxial or biaxial in-plane loads involving compression. This is due to the possibility of the edge correction terms, those involving exponents of γ_x or γ_y, becoming oscillatory. The author became aware of this problem shortly after the publication of the Note under discussion.[1] He has since established, however, that if a modified version of the edge effect method (originally proposed by Vijayakumar[2] and Elishakoff[3] is used, then problems for which γ_x and/or γ_y are real or imaginary can be treated satisfactorily.

A Technical Note[4] on the application of the modified edge effect method to the buckling and vibration of in-plane loaded plates, in which is included an example for which an edge correction term would become oscillatory, has already been submitted to the Journal.

References

[1] Dickinson, S.M., "Bolotin's Method Applied to the Buckling and Lateral Vibration of Stressed Plates," *AIAA Journal*, Vol. 13, Jan. 1975, pp. 109-110.

[2] Vijayakumar, K., "A New Method for Analysis of Flexural Vibration of Rectangular Orthotropic Plates," *Journal of the Aeronautical Society of India*, Vol. 23, No. 4, 1971, pp. 1974-204.

[3] Elishakoff, I.B., "Vibration Analysis of Clamped Square Orthotropic Plate," *AIAA Journal*, Vol. 12, July 1974, pp. 921-924.

[4] Dickinson, S.M., "Modified Bolotin's Method Applied to Buckling and Vibration of "Stressed Plate," *AIAA Journal*, Vol. 13, Dec. 1975, to be published.

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